Waveform Modeling & Option Analysis For Radar Systems Using MATLAB

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Need of **Waveform Modeling & Option Analysis**

**Radar System Design Flow**

**Operational Scenario & Operational Requirements**

**Waveform Selection & Design**

- Antenna-Radome Subsystem Design
- RF/MW Transceiver Subsystem Design
- DSP & Power Supply Subsystem Design
- Integrated System Test & Evaluation

**Waveform Modeling & Option Analysis**

- Maximum Unambiguous Range & Doppler
- Range & Doppler Resolution
- Required Tx Power
- Required Update Rate
- LPI or LPD
### Waveform Modeling & Option Analysis

#### Dependency of System Parameter on Waveform Design

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Unambiguous Range &amp; Doppler Tolerance</td>
<td></td>
</tr>
<tr>
<td>Blind Range</td>
<td></td>
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<tr>
<td>Range Resolution</td>
<td></td>
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<tr>
<td>Doppler Resolution</td>
<td></td>
</tr>
<tr>
<td>Required $P_{\text{average}}$ for Max. Range Detection &amp; Measurement Update Time</td>
<td></td>
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<tr>
<td>ECCM Features</td>
<td></td>
</tr>
<tr>
<td>Ambiguity Plots &amp; Contours For Specific Type of Waveform</td>
<td></td>
</tr>
<tr>
<td>Pulse Width or Period in case of Duty Cycle &lt; 100 %</td>
<td></td>
</tr>
<tr>
<td>Bandwidth, Pulse Width</td>
<td></td>
</tr>
<tr>
<td>PRI, Period, Integration Time</td>
<td></td>
</tr>
<tr>
<td>PRI, Period, Duty Cycle, Coherent &amp; Non-coherent Integration Time</td>
<td></td>
</tr>
<tr>
<td>Low Probability of Intercept or Low Probability of Detection</td>
<td></td>
</tr>
</tbody>
</table>
The choice of transmitter waveform depends on the following factors:

- Ease of generation of the waveform
- Peak power required by the waveform
- Extent of compression offered by the waveform and the resultant resolution
- Doppler tolerance and Range–Doppler ambiguity relation.
- The extent to which the waveform can be detected, intercepted, jammed.

The waveforms considered for analysis:

- Pulsed RF
- Linear Frequency Modulated Waveform.
- Pseudo Randomly Bi-phase Modulated Continuous Waveform
Importance of Ambiguity Plots & Contours
For Waveform selection

**Ideal Ambiguity Diagram**

\[ | \chi(t_d, f_d) |^2 \]

**Unattainable**

**An Approximation to Ideal Ambiguity Diagram**

\[ |x(t_d, f_d)|^2 \]

\[ T=\text{Signal Duration} \]
\[ B=\text{Signal Bandwidth} \]
\[ E=\text{Energy of signal} \]

**Properties:**

- Max. value of \( |x(t_d, f_d)| = |x(0,0)|^2 = 2E^2 \)
- \( |x(-t_d, -f_d)|^2 = |x(t_d, f_d)|^2 \)
- Along \( td \) Axis \( |x(t_d, f_d)|^2 \) is a autocorrelation function of \( u(t) \)
- Along \( fd \) Axis \( |x(t_d, f_d)|^2 \) is a Spectrum of \( u(t) \)
- Total volume the ambiguity plot is constant: \( v = 2E^2 \)
Ambiguity Plots & Contours For LFM & PN Coded Waveform

**Knife Edge Ambiguity function**

\[ |\chi(t_d, f_d)|^2 \]

**Thumb Tack Ambiguity function**

\[ \frac{1}{B} \]

\[ \frac{1}{T} \]

\[ \frac{1}{BT} \]
MATLAB Simulation Results For Pulsed RF Signal
Time Domain Plot: Coherent Pulse Train

Ton = 40 nsec, PRI = 200 nsec, DC = 20 %
Signal Spectrum: Coherent Pulse Train

Ton = 40 nsec, PRI = 200 nsec, DC = 20%

Spacing between Spectral Lines = PRF = 5 MHz
Time Domain Plot: Coherent Pulse Train

Ton = 400 nsec, PRI = 1000 nsec, DC = 40%
Signal Spectrum: Coherent Pulse Train

$T_{on} = 400 \text{ nsec}, \ PRI = 1000 \text{ nsec}, \ DC = 40\%$

Spacing between Spectral Lines: $PRF = 1 \text{ MHz}$
Ambiguity Plot: Coherent Pulse Train

Coherent Pulse Train: $T_p=0.4\,\text{usec}, \, PRI=1\,\text{usec}, \, N=3, \, T^*B=1$
MATLAB Simulation Results
For
Linear Frequency Modulated Signal
LFM Continuous Waveform
Saw-tooth & Triangular Sweeps

Amplitude

Frequency

Time

Sweep Bandwidth ($\Delta F$)

Sweep Period ($T_m$)

Up-Sweep Period

Down-Sweep Period

Sweep Frequency B.W. ($\Delta F$)
Ambiguity Plots of Single Pulse with Different Pulse Widths

Ambiguity Plot
Pulse Width ($T_p$) = 1 µsec

B.W. (B) = $1/T_p$

Ambiguity Plot
Pulse Width ($T_p$) = 50 µsec

Contour Plot: $T_P = 1$ µsec

Contour Plot: $T_P = 50$ µsec
Ambiguity Plots of LFM Pulse with Different TxB Products

- $T_P = 0.5 \, \mu\text{sec}, \; B = 10\,\text{MHz}$, $TxB = 5$
  - Slope of Ridge = $B/T = 20\,\text{MHz/\mu sec}$

- $T_P = 0.5 \, \mu\text{sec}, \; B = 20\,\text{MHz}$, $TxB = 10$
  - Slope of Ridge = $B/T = 40\,\text{MHz/\mu sec}$

$B \gg 1/T_p$ & $T*B \gg 1$
Amplitude Spectra: LFM Chirp
Effect of Sweep Frequency Bandwidth (B)

**LFM Up Chirp: Amplitude Spectra: Bandwidth = 500 MHz**

Y-Axis is in normalized scale

**LFM Up Chirp: Amplitude Spectra: Bandwidth = 100 MHz**
Effect of (Time x Bandwidth) Product on LFM Autocorrelation

- **T x B = 20**
- **T x B = 50**
- **T x B = 500**
Range–Doppler Coupling Effect in LFM Waveform

\[ \alpha = \text{Freq. Sweep Rate} \]

Up Chirp: \[ \alpha = \frac{+B}{T} \]

Down Chirp: \[ \alpha = \frac{-B}{T} \]

Shift in peak of Matched Filter Output
\[ T = \frac{fd \times T}{B} \]

Here, \( \alpha = 1 \text{ MHz/1 usec} \)

Compressed Pulse shape & SNR are Doppler Tolerant
Ambiguity Plot: LFM Pulse Train

LFM pulse train, T*B = 4, N=3 pulses

Contour Plot
MATLAB Simulation Results
For
Pseudo-Randomly Bi-phase Coded Waveform
Pseudo-Randomly Bi-phase Coded Waveform

+1 + + + +
-1 - - -

-18 dB
-36 dB

\[ \left( \frac{\sin \pi f_t \tau_b}{\pi f_t \tau_b} \right)^2 \]

\[ \frac{1}{\tau_b} = 5 \text{ MHz} \]
\[ \frac{1}{\tau_\omega} = 80 \text{ kHz} \]

- RF CENTER FREQUENCY
- PRC BIT WIDTH (200 nsec)
- \( \tau_\omega = L \tau_b = 12.6 \mu \text{sec} \)
- \( L = 63 \text{ PRC BITS/WORD} \)
Pseudo-Randomly Bi-phase Coded Waveform

Time Domain Plot

Carrier Signal ($f_c = 1$ GHz)

Modulating PN Code ($L = 7$ bits and Chip Width ($T_b$) = 12.5 nsec)

Bi-phase Modulated Signal
Amplitude Spectra
Pseudo-randomly Bi-phase Coded Waveform

Single Sided Amplitude Spectra:
Chip Width = 25 nsec

Single Sided Amplitude Spectra:
Chip Width = 10 nsec
Stem Plot
Amplitude spectra of Biphase coded waveform

Power Spectra: L = 63 Bits, Tb = 20 nsec

$1/T_w = 1/(1.26 \mu s) = 793.65 \text{ KHz}$

Y-Axis in normalized scale

Code Length, $L = 63$
Chip Width, $T_b = 20 \text{ nsec}$
Continuous Plot
Amplitude spectra of Biphase coded waveform

Y-Axis in dB

Power Spectra: L=63 Bits, Tb=20 nsec

20 \log \left( \frac{1}{L} \right) 

10 \log \left( \frac{1}{L} \right)

Code Length, L = 63
Chip Width, Tb = 20 nsec
Comparison of Ambiguity Plots: PN Codes

Effect Of Code Length

PN Code: \( L=15, \; T^*B=15 \)

- Volume under the peak is less
- Width of central response is less
- Height of Plateu=\( E^2/B*T \)

PN Code: \( L=63, \; T^*B=63 \)
Contour Plots: PN Codes

Effect of Code Length

PN CODE: \( L=15, \) \( TxB = 15 \)

PN CODE: \( L=63, \) \( TxB = 63 \)
Zero Doppler cut of Ambiguity plots of PN Codes
Effect Of Code Length

PN Code: \( L=15, T\cdot B=15 \)

PN Code: \( L=63, T\cdot B=63 \)
Comparison of Range Sidelobes of PN CODED & LFM Waveforms

Zero Doppler Cut of PN Coded Continuous Waveform
L=127, TxB =127

Zero Doppler Cut of LFM Waveform
Tp =1µsec, B = 127MHz , T*B=127
Comparison of Autocorrelation Properties
Effect of Different Type of Codes

Auto Correlation of Barker Code: 13 bits: Normalized o/p

Auto Correlation of Kasami Code: 63 bits: Normalized o/p

Auto Correlation of Gold Code: 63 bits: Normalized o/p

Auto Correlation of PN Code: 63 bits: Normalized o/p
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<th>FMCW</th>
<th>PN Coded CW</th>
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<td><strong>Tx Peak Power Requirement</strong></td>
<td></td>
<td></td>
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<td><strong>Range Resolution</strong></td>
<td>$\Delta R = C \times \frac{Tp}{2}$</td>
<td>$\Delta R = \frac{C}{2} \times \Delta F$</td>
<td>$\Delta R = C \times \frac{Tb}{2}$</td>
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<td>$Tp=$Pulse Width</td>
<td>$\Delta F=$Frequency Sweep</td>
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<td>Higher range Resolution can be achieved easily.</td>
<td>Limitation on low value of Tb due to ADC technology.</td>
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<td>Higher $T \times B$ Product by increase in $B = \Delta F$.</td>
<td>Higher $T \times B$ Product by increase in Bandwidth $= \frac{1}{Tb}$.</td>
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<td><strong>Long Range Measurement</strong></td>
<td>Higher PRI.</td>
<td>Longer Waveform Period($T$).</td>
<td>Longer Waveform Period($Tw$).</td>
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<td>Low Duty Cycle.</td>
<td>100% Duty Cycle.</td>
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<td>Req. Of High Peak Power</td>
<td>Lower sweep rate($\Delta F/T$)</td>
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<td>Higher $T \times B = T \times \Delta F$ Product by increase in $T$.</td>
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**Results:** Comparative Evaluation of Radar Waveforms

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<td><strong>T x B Product</strong></td>
<td>T x B = Tp x 1/ Tp = 1 (Unity)</td>
<td>T x B = T x ΔF &gt;&gt; 1 Δ F = Frequency Sweep</td>
<td>T x B = Tw x 1/Tb = L * Tb/Tb = L L = Code length (no. of bits) Tb = Chip Width Lower than FMCW Radar.</td>
</tr>
<tr>
<td></td>
<td>Tp = Pulse Width</td>
<td>T = Period, ΔF = Bandwidth</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T = Period, B = Bandwidth</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Time Side lobes</strong></td>
<td>Very High</td>
<td>Lower.</td>
<td>Lowest.</td>
</tr>
<tr>
<td></td>
<td>_req. of Weighing filter following the Match filter.</td>
<td>Req. of Weighing filter following the Match filter.</td>
<td>No req. of Weighing filter following the Match filter.</td>
</tr>
<tr>
<td></td>
<td>Use of Weighing filter results in reduced SNR.</td>
<td>Use of Weighing filter results in reduced SNR.</td>
<td>Higher SNR achieved. Better Correlation achieved.</td>
</tr>
<tr>
<td><strong>Doppler Tolerance</strong></td>
<td>Good.</td>
<td>Supports doppler shift upto +/- B/10.</td>
<td>Fdmax = 1/(4 * Tw)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Time side lobes performance remains intact with large doppler shifts.</td>
<td>Time side lobes performance &amp; the peak of correlation deteriorates with large doppler shifts.</td>
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<tr>
<td>Spill over &amp; Tx Leakage</td>
<td>Less</td>
<td>Poor Performance.</td>
<td>Overall performance improvement by 15-20dB compared to FMCW.</td>
</tr>
<tr>
<td></td>
<td>Due to discontinuous waveform pattern.</td>
<td>Operation limited to short range applications</td>
<td>Decrease in spill over by 1/L^2 (i.e. 48 dB for L=127) compared to unmodulated radar signal.</td>
</tr>
<tr>
<td>Range Doppler Coupling</td>
<td>No.</td>
<td>Yes.</td>
<td>No.</td>
</tr>
<tr>
<td></td>
<td>Due to the Frequency Sweep Rate (ΔF/T).</td>
<td></td>
<td></td>
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<tr>
<td>Low Probability of Intercept &amp; ECCM</td>
<td>No LPI &amp; ECCM features.</td>
<td>Better than Pulse delay ranging due to Pulse Compression.</td>
<td>Better LPI &amp; ECCM capability compared to FMCW.</td>
</tr>
</tbody>
</table>
Thank You!