3-D Finite Element Analysis with MATLAB

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Agenda

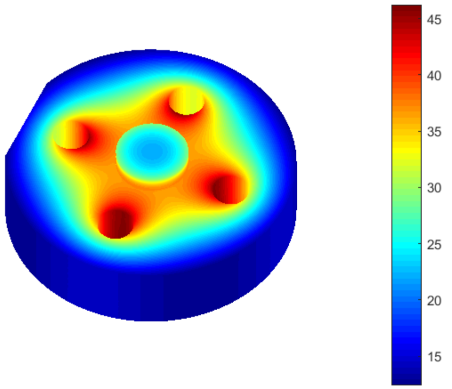
- Motivation
- Finite Element Analysis (FEA) with MATLAB
- 3-D FEA workflow with PDE Toolbox
- Examples
- Conclusion
- Resources

Motivation

- Applications in Aerospace, Automotive, Energy industries often require
  - High fidelity modeling of structural mechanics, heat transfer, electromagnetics problems etc.
    - Finite Element Analysis (FEA): popular for solving underlying Partial Differential Equations (PDE)
    - Accurately capture physics with 3-D FEA

Motivation

- Applications in Aerospace, Automotive, Energy industries often require
  - High fidelity modeling of structural mechanics, heat transfer, electromagnetics problems etc.
    - Finite Element Analysis (FEA): popular for solving underlying Partial Differential Equations (PDE)
    - Accurately capture physics with 3-D FEA
- 3-D FEA will let you answer questions such as



NumHoles	HolesRadius	AvgTempVariable	InputForTargetTemp	MaxMinSpread	OperatingCost
4	1.2	1.9545	4.6048	0.7581	18.419
4	1.1	1.8816	4.7831	0.60484	19.133
4	1	1.8197	4.9458	0.49596	19.783
3	1.2	1.3607	6.614	0.69789	19.842

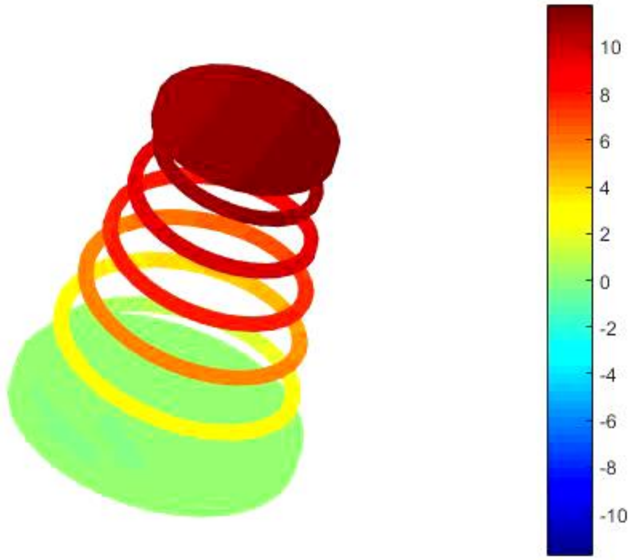
What is optimal #holes, radius for efficient heating?

3-D FEA with MATLAB

- Typical tasks with 3-D FEA and MATLAB are
  - Pre-processing, Post-processing and visualization, Automation/scripting and Simulation

3-D FEA with MATLAB: PDE Toolbox

- Typical tasks with 3-D FEA and MATLAB are
  - Pre-processing, Post-processing and visualization, Automation/scripting and Simulation
- Release 15a brings 3-D FEA to PDE Toolbox

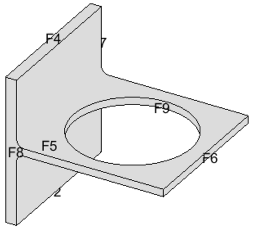


3-D FEA with MATLAB: Other Options

- MATLAB integration with commercial FEA packages w/ direct or 3rd party. Examples
  - COMSOL w/ LiveLink for MATLAB
  - ANSYS
    - Mechanical
    - Fluent
    - Maxwell
    - HFSS
  - Femlink + Structural Dynamics Toolbox from SDTOOLS
  - IMAT from ATA
- Community contributions on MATLAB File Exchange

Partial Differential Equations (PDE) Toolbox

- Solve coupled PDEs in 2-D and 3-D space using the Finite Element Method
- Workflow
  1. Define geometry



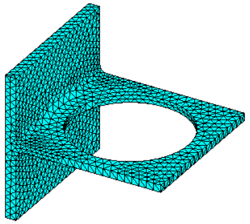
  2. Define equations (PDE coefficients)

$$\mathbf{d}\frac{\partial u}{\partial t} - \nabla \cdot (\mathbf{c} \otimes \nabla u) + \mathbf{a}u = \mathbf{f}$$

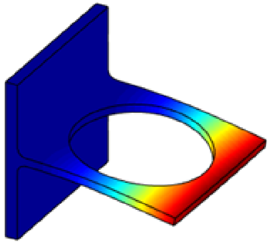
  3. Define boundary conditions, initial conditions

$$\mathrm{n} \cdot \mathbf{c} \otimes \nabla u + \mathbf{q}u = \mathbf{g}$$

  4. Mesh



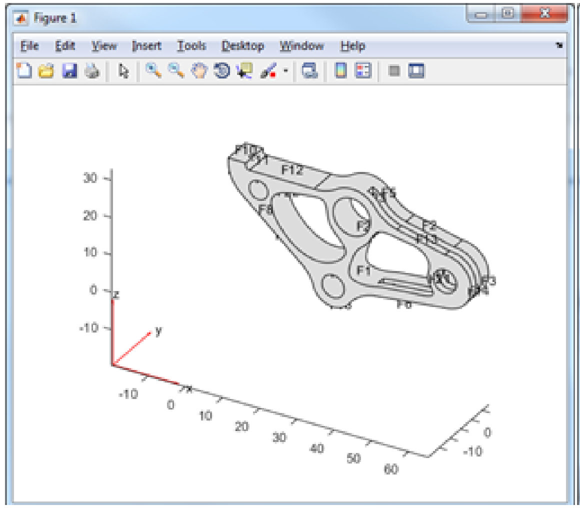
  5. Solve and visualize results



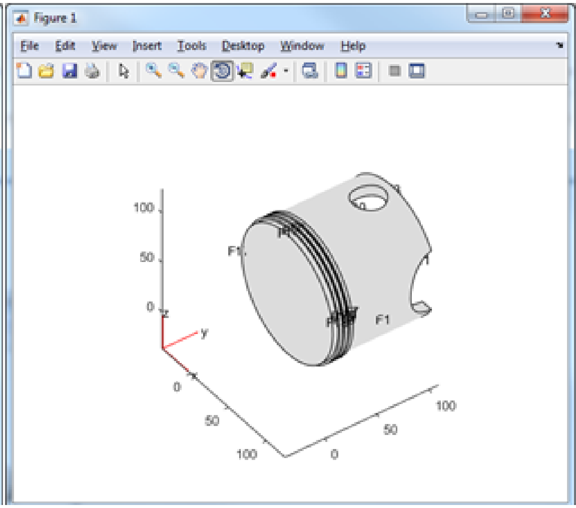
Step 1 – Define Geometry

- Import geometry from **ST**ereo**L**ithography (STL) file
  - Almost all CAD software can export geometry to STL files

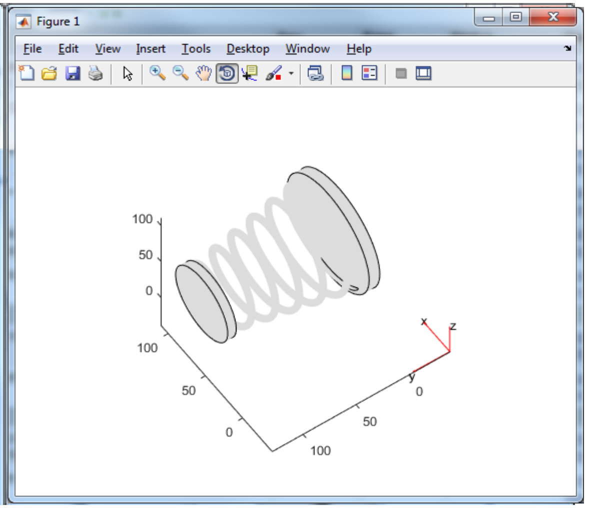
Bellcrank



Automobile Piston



Helical Tapered Spring



Step 2 – Define Equations (PDE Coefficients)

- Types of PDEs supported:

Type	Description	Example Applications
Elliptic	$-\nabla \cdot (\mathbf{c} \otimes \nabla u) + \mathbf{a}u = \mathbf{f}$	electrostatic, magnetostatic, heat conduction, piezoelectric
Parabolic	$\mathbf{d}\frac{\partial u}{\partial t} - \nabla \cdot (\mathbf{c} \otimes \nabla u) + \mathbf{a}u = \mathbf{f}$	heat transfer (diffusion), reaction-diffusion
Hyperbolic	$\mathbf{d}\frac{\partial^2 u}{\partial^2 t} - \nabla \cdot (\mathbf{c} \otimes \nabla u) + \mathbf{a}u = \mathbf{f}$	wave, structural dynamics
Eigenvalue	$-\nabla \cdot (\mathbf{c} \otimes \nabla u) + \mathbf{a}u = \lambda \mathbf{d}u$	structural mode shapes

- Coefficients can
  - represent coupled PDEs
  - be functions of $(u, t, ux, uy, uz, x, y, z) \Rightarrow$ support for nonlinear problems
  - be complex numbers

Step 2 – Define PDE Coefficients

- Map your PDE to PDE coefficient form
  - Example: For linear isotropic elasticity, the "c" coefficient in $-\nabla \cdot (\mathbf{c} \otimes \nabla u) + \mathbf{a}u = \mathbf{f}$ looks like
  - Type equation here.

$$c = \begin{bmatrix} c_1 & 0 & 0 & 0 & c_{12} & 0 & 0 & 0 & c_{12} \\ \bullet & G & 0 & G & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & G & 0 & 0 & 0 & G & 0 & 0 \\ \bullet & \bullet & \bullet & G & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & c_1 & 0 & 0 & 0 & c_{12} \\ \bullet & \bullet & \bullet & \bullet & \bullet & G & 0 & G & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & G & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & G & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & c_1 \end{bmatrix}$$

$$G = \frac{E}{2(1+\nu)}$$

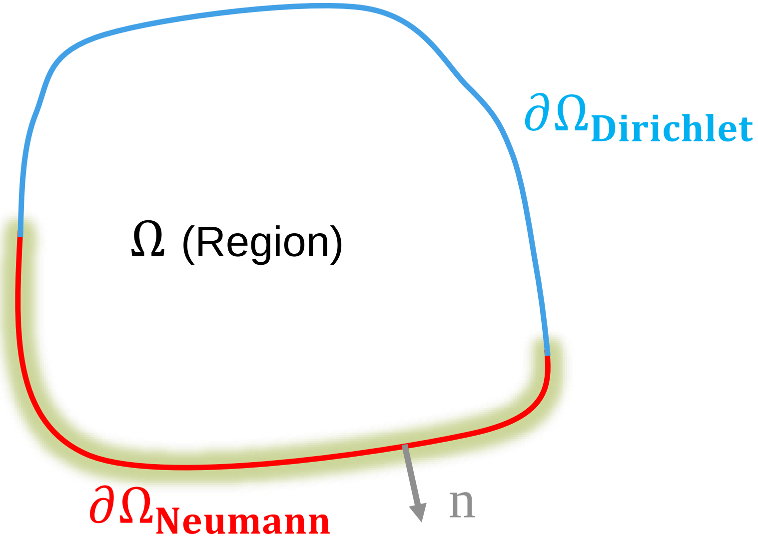
$$c_1 = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

$$c_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

- Coefficients can be specified via scalars, expressions, functions

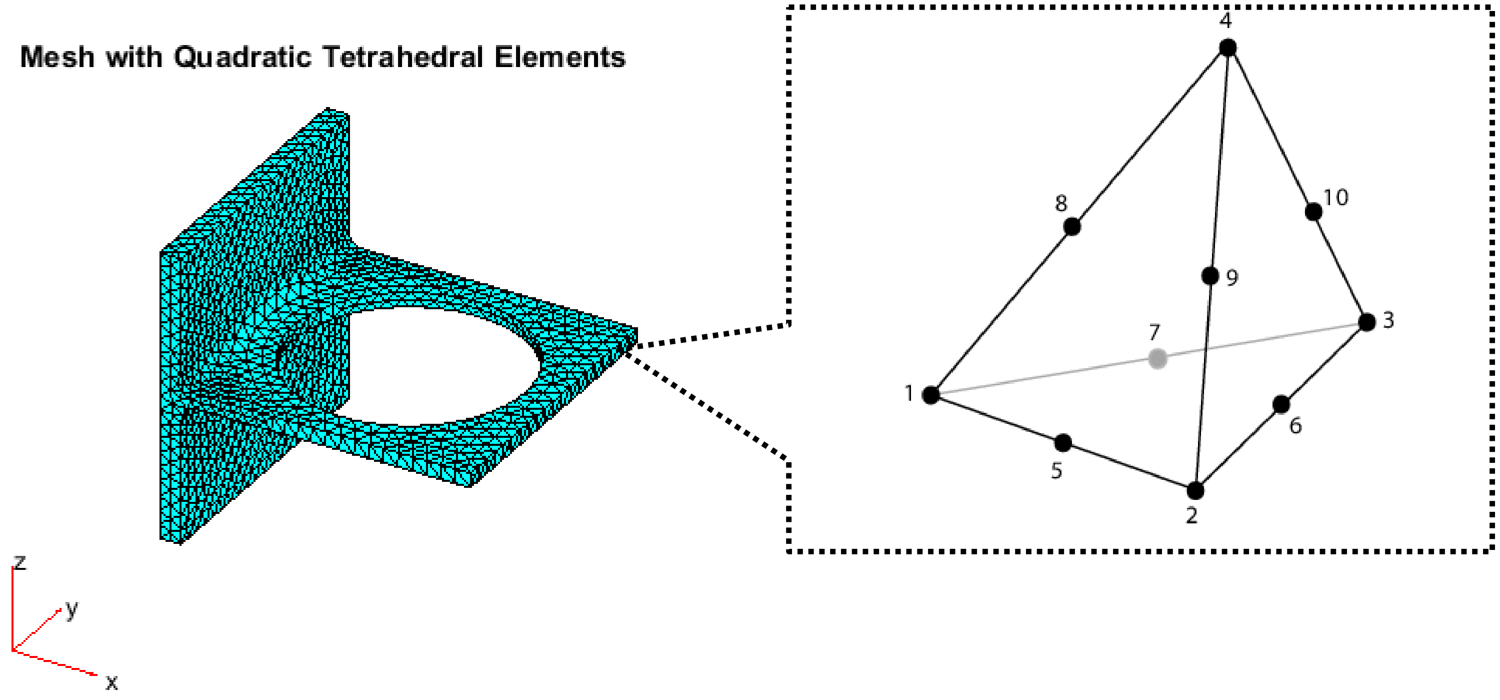
Step 3 – Define Boundary Conditions

Type	Description	Example
Dirichlet	$\mathbf{h}u = \mathbf{r}$	Zero potential on boundary
Generalized Neumann (Robin)	$\mathrm{n} \cdot \mathbf{c} \otimes \nabla u + \mathbf{q}u = \mathbf{g}$	Fixed flux on boundary



Step 4 – Mesh

- Mesh Geometry

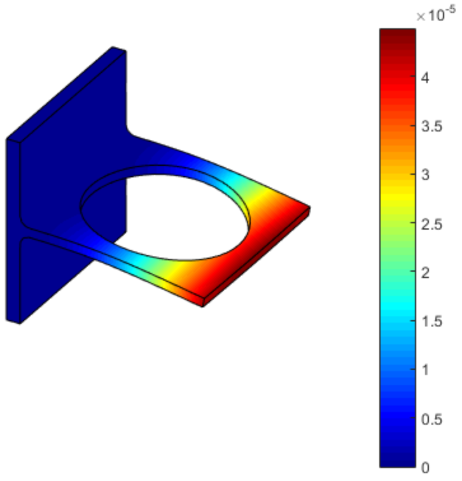


- Uses tetrahedral elements
- Choice of quadratic (default) or linear basis

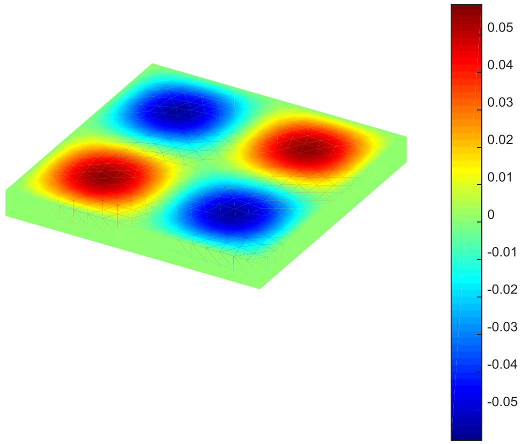
Step 5 – Solve and Visualize Results

- Use static, time-domain (using Method-of-Lines), eigenvalue problem, nonlinear solvers
- Visualize results
- Interpolate solution to any interior point

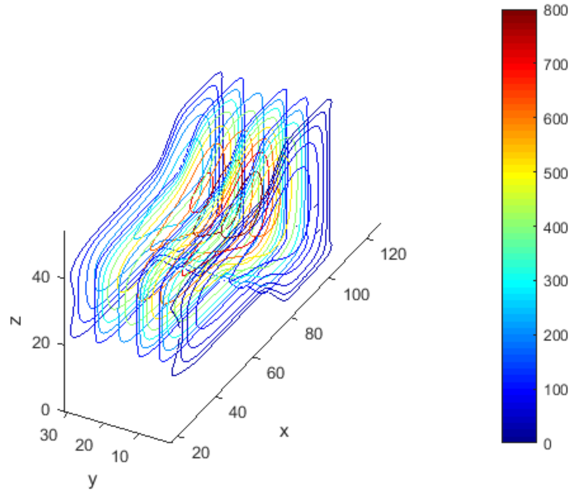
Static solution plot



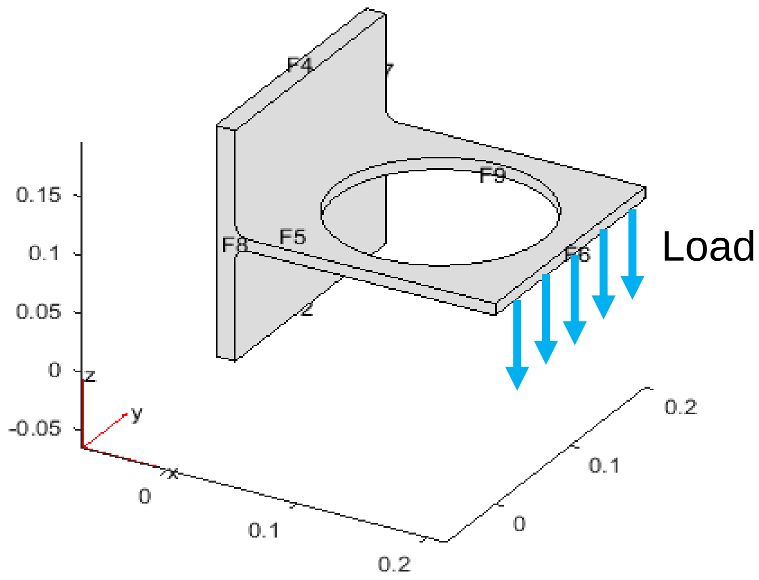
Mode plot



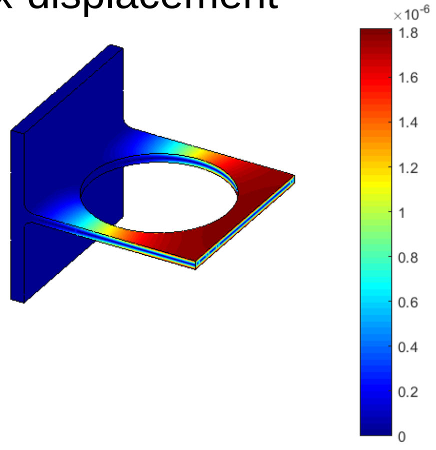
Sliced contour plot



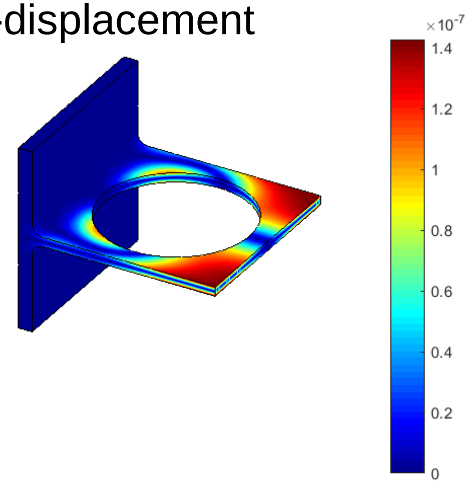
Example – Strained Bracket Deflection



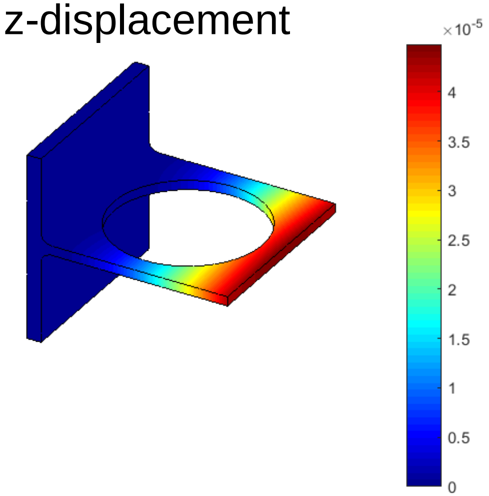
x-displacement



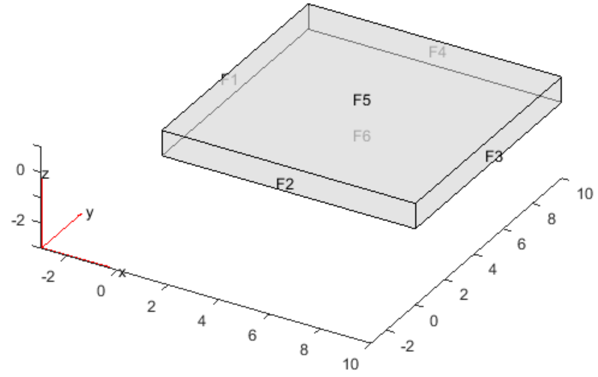
y-displacement



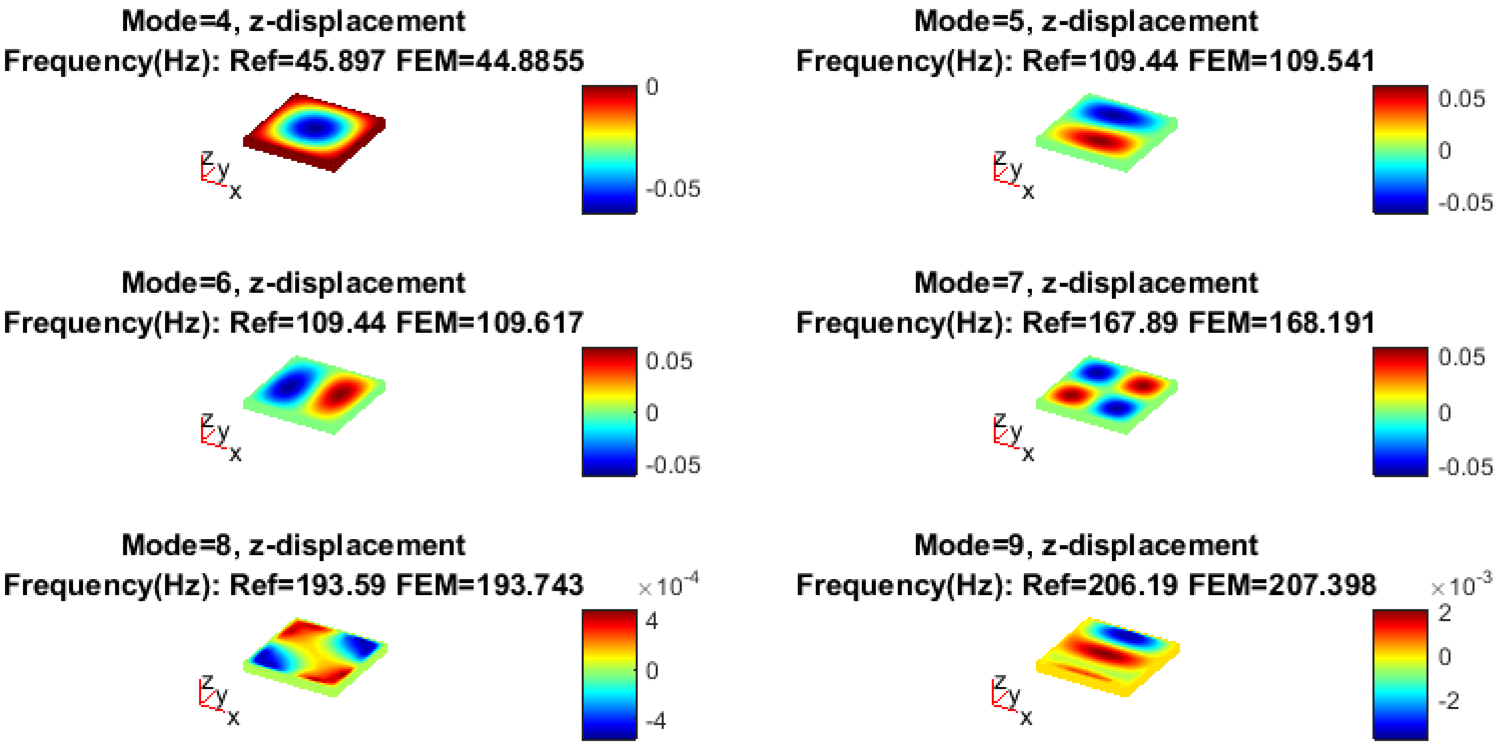
z-displacement



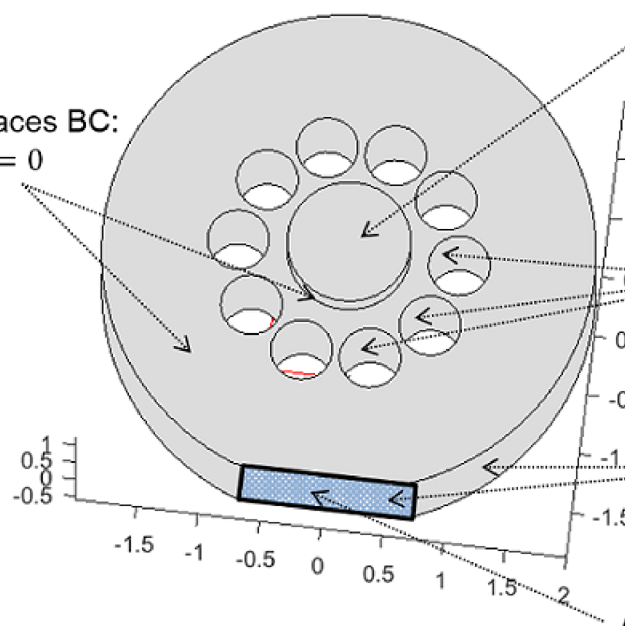
Example – Modal Analysis of Plate



First 3-modes: rigid body modes



Example – Optimal Geometry for Target Average Temperature – A Parametric Study



<u>Objective</u>: Find geometry that achieves target AvgTemp with

a) smallest total Input (operating cost)

b) smallest MaxMinSpread (temp. variation)

Conclusions

- 3-D FEA with PDE Toolbox lets you accurately model your geometry and capture the underlying physics in MATLAB
  - Solve common and custom 3-D PDEs
- Perform custom post-processing and visualization
- Share reports of your FEA based studies
- Use 3-D FEA as part of your workflow in MATLAB

Learn More: 3-D FEA with MATLAB

mathworks.com/products/pde