**Exercises: Numerical Differentiation**

**Problem 1:** Use the ***diff*** command in MATLAB to find the first and second derivative of the following function with respect to t. Include both your results and your MATLAB commands.

**Problem 2:** The graph below shows position measurements (in cm) collected every 0.5 seconds over a 10 second interval of time.



1. Using a Δt = 0.5 sec, estimate the velocity at t =  sec using the 2-point estimate and the 3-point estimate for derivative. **Be sure to show your work and include units!**

2-point Estimate of Velocity at t = 3 sec: \_\_\_\_\_\_\_\_\_

3-point Estimate of Velocity at t = 3 sec: \_\_\_\_\_\_\_\_\_

1. Using the estimate for 2nd derivative and a Δt = 0.5 sec, estimate the acceleration at t = 3 sec. Again, show work!

Estimate of Acceleration at t = 3 sec: \_\_\_\_\_\_

1. What could be changed to improve the accuracy of the derivative estimates?

**Problem 3:** The following equation describes underdamped harmonic motion:

**d:** displacement of the mass in meters (m)

**d0:** initial displacement of the mass in meters (m)

**ω:** the frequency of oscillation (rad/s)

**M:** mass (kg)

**K:** spring constant (N/m)

**B:** damping coefficient (N**.**s/m)

(a) Write a program in MATLAB that:

* Prompts the user to enter a value for dt
* Creates variables K, B, and M, and sets them to the following values:

K = 200; B = 4; M = 0.5;

* Calculates α and ω.
* Calculates a range of t-values starting at 0, incrementing by dt, and ending at 5/α
* Creates a vector of displacement measurements at the t-values (assume d0 = 1).
* Uses the displacement measurements to estimate the velocity using the 3 PT estimate for derivative
* Uses the displacement measurements to estimate acceleration
* Creates another set of t-values with a much finer increment than dt
* Calculates displacement, actual velocity, and actual acceleration over this second range of t-values.
* Uses subplot to plot displacement in one sub-window, velocity and the velocity estimate in a second sub-window, and acceleration and acceleration estimate in a third sub-window.
* Includes titles and labels (with appropriate units) for each sub-window.

1. Run your program using dt = 0.1 and paste the resulting figure below.
2. Run your program using dt = 0.025 and paste the resulting figure below.
3. Compare the two graphs in terms of accuracy of the estimates. What do you notice about the accuracy of the estimates around t = 0 and the accuracy of the estimates closer to the end time? Why does this occur?
4. Why is 5/α a good end time for calculating and plotting the displacement?